Chapter 10 / **Example 21**

# Application of differentiation to kinematics

The height (m) of a rocket projected vertically into the air until it returns to the ground is represented by the function $h(t) = -0.11t^2 + 1.32t + 1.5, t \geq 0$ where $t$ is the number of seconds after the rocket was launched.

**a** State the height at which the rocket was launched.

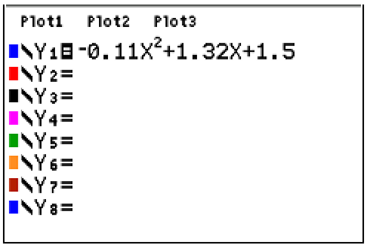
**b** Find the maximum height reached by the rocket.

**c** Calculate the velocity of the rocket at $t = 7.5$ s and state whether it is ascending or descending at this time.

**d** Find the other time at which the rocket is travelling at the same speed as when. $t = 7.5$ s

Press [F1] [Y=] to display the equation entry screen.

Type - $0.11x^2$ + $1.32x$ + 1.5 and press [ENTER] to enter the equation as $Y_1$.

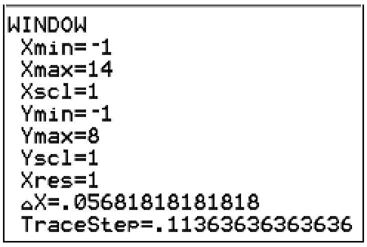



Choose appropriate axes to show the graph.

Press [F2] [WINDOW].

Set the axes to show - 1 £ $x$ £ 14 and - 1 £ $y$ £ 8 with a scales of 1.

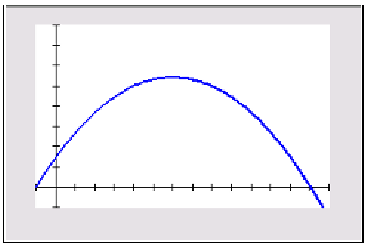
You can leave the other items as they are.




Press [F5] [GRAPH] when you have finished.

The GDC displays the graph $Y_1$.

The default axes are - 10 £ $x$ £ 10 and - 10 £ $y$ £ 10 .

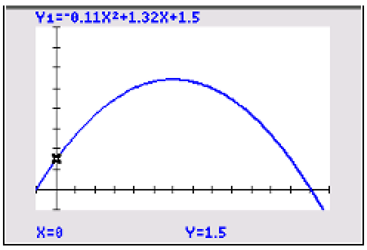


To find the $y$-intercept press [2nd] [F4] [CALC] 1:value

Press Ê [ENTER] to change the $x$-coordinate to 0.

The $y$-intercept is at $(0, 1.5)$.

The rocket was launched from a height of 1.5 m.



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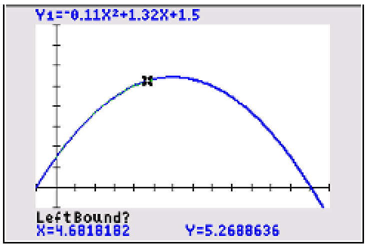
To find the vertex press [2nd] [F4] [CALC] 4:maximum

You will need to give the left and right bounds of the region that includes the vertex.

The GDC shows point on the curve and asks you to set the left bound. Move the point using ~ | and choose a position to the left of the vertex.

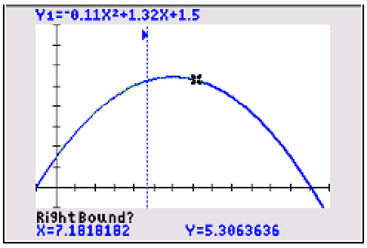
Press [ENTER].




The GDC shows a line where you have set the left bound and a point on the curve.

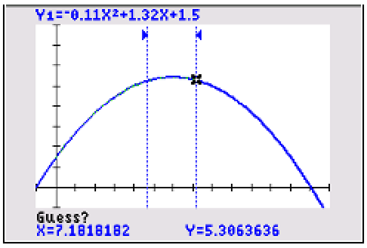
Move the point using ~ | and choose a position to the right of the vertex.

When the region contains the vertex, Press [ENTER].




The GDC requires an initial guess for the position of the zero. Choose the default position.

Press [ENTER].

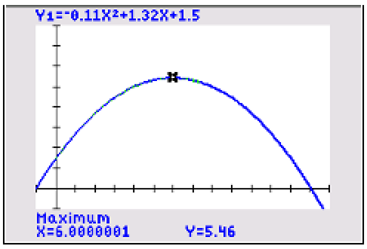



The GDC displays the vertex.

The vertex of the quadratic function is at $(6, 5.46)$.

*Take care to interpret what the GDC displays. 6.0000001 is very close to 6. The small difference is due to the numerical way that the GDC calculates the value.*

The maximum height is 5.46 m.




To find the velocity of the rocket with a GDC, use the first derivative.

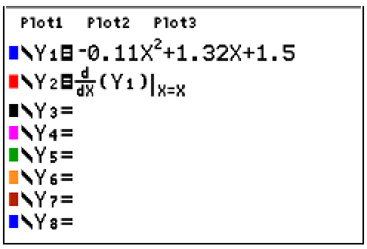
Press [F1] [Y=] to display the equation entry screen.

Press [ALPHA] [F2] 3:nDeriv

The template has spaces for the variable, $x$, the function and the value that it is evaluated at.

Enter X in the denominator and the function $Y_1$ using [ALPHA] [F4] 1:$Y_1$

Type X and press [ENTER].



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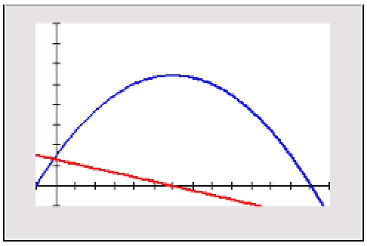
Press [F5] [GRAPH] to display the graph screen.

The GDC displays the graphs $Y_1$ and its first derivative.



To find the velocity at $t$ = 7.5 press [2nd] [F4] [CALC] 1:value

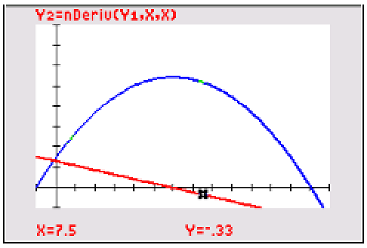
Type 7.5 and press [ENTER] to change the $x$-coordinate to 7.5.

Use [ENTER] to select a point on $Y_2$.

The point is $(7.5, -0.33)$.

The velocity is $-0.33$ ms$^{-1}$.

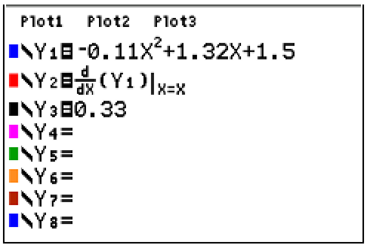
Because the velocity is negative, the height is decreasing and so the rocket is descending.




To find another point where $v$ = +0.33 ms$^{-1}$ draw the line $y$ = 0.33 and find the point of intersection.

Press [F1] [Y=] to display the equation entry screen.

Type 0.33 and press [ENTER] to enter the equation as $Y_3$.

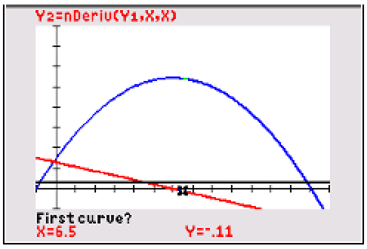



Press [F5] [GRAPH] to display the graph screen.

Press [2nd] [F4][CALC] 5:intersect

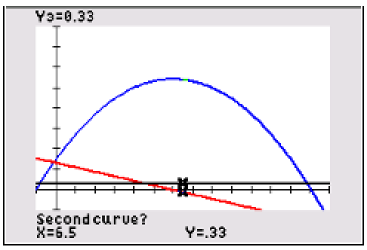
To find the intersection you need to choose the two lines that intersect.

The GDC shows a cross on one of the lines and 'First curve?'. Choose $Y_2$ and press [ENTER].




The GDC shows a cross on the other line and 'Second curve?'.

Press [ENTER].



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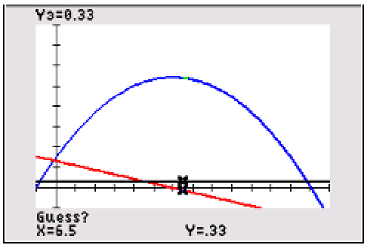
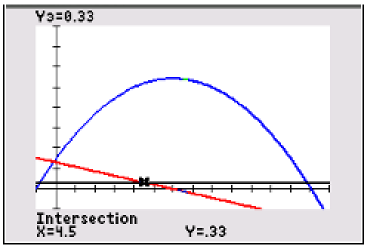
| | |
|---|---|
| The GDC requires an initial guess for the position of the intersection. Choose the default position.<br>Press ENTER. |  |
| The GDC displays the intersection of the two straight lines at the point $(4.5, 0.33)$<br>The rocket has the same speed at $t$ = 4.5 s. |  |